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ON NANO (1,2)* SEMI-GENERALIZED CLOSED SETS IN NANO BITOPOLOGICAL SPACES

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ABSTRACT

The purpose of this paper is to define Nano Bitopological space and study a new class of sets called Nano (1, 2)* semi-generalized closed sets in Nano Bitopological spaces. Basic properties of Nano (1, 2)* semi-generalized closed sets are analyzed. Also the new Characterization on Nano (1, 2)* semi-generalized spaces are introduced and their relation with already existing well known spaces are also investigated.

KEYWORDS: Nano (1, 2)* Open Sets, Nano (1, 2)* Closed Sets, Nano (1, 2)* Closure, Nano (1, 2)* Interior, Nano (1, 2)* Semi Closed Sets, Nano (1, 2)* Semi-Closure, Nano (1, 2)* Semi-Interior, Nano (1, 2)* Semi-Generalized Closed Sets, Nano (1, 2)* Semi-T_0, Nano (1, 2)* Semi-$T_{1/2}$, Nano (1, 2)* Semi-T_1

1. INTRODUCTION

In 1970, Levine [11] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. While in 1987, P.Bhattacharyya et.al. [1] Have introduced the notion of semi generalized closed sets in topological spaces. In 1975, S.N.Maheshwari et al., [12] have defined the concepts of semi separation axioms. The notion of nano topology was introduced by Lellis Thivagar [8]. In 1963, J.C.Kelly [7] initiated the study of bitopological spaces. Mean while in 1987, Fukutake [4] introduced generalized closed sets and pairwise generalized closure operator in bitopological spaces. In 1989, Fukutake [5] introduced semi open sets in bitopological spaces. In 2014, K. Bhuvaneswari et al., [3] have introduced the notion of nano semi generalized and nano generalized semi closed sets in nano topological space. In this paper, the concept of new class of sets on nano bitotpological spaces called nano (1, 2)* semi generalized closed sets and the characterization of nano (1, 2)* semi generalized spaces are introduced. Also study the relation of these new sets with the existing sets.

2. PRELIMINARIES

Definition 2.1 [10]: A subset A of a topological space (X,τ) is called a semi open set if $A \subseteq cl[Int(A)]$. The complement of a semi open set of a space X is called semi closed set in X.

Definition 2.2 [1]: A semi-closure of a subset A of X is the intersection of all semi closed sets that contains A and it is denoted by scl (A).

Definition 2.3 [1]: The union of all semi open subsets of X contained in A is called semi-interior of A and it is denoted by $sInt(A)$.

Definition 2.4 [1]: A subset A of (X,τ) is called a semi generalized closed set (briefly sg-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.

Definition 2.5 [6]: A space (X,τ) is called T_0 space, if and only if, each pair of distinct points x, y of X, there exist a open set containing one but not the other.

Definition 2.6 [6]: A space (X,τ) is called $T_{1/2}$ space, if and only if, every generalized closed set is closed.

Definition 2.7 [6]: A space (X,τ) is called T_1 space, if and only if, each pair of distinct points x, y of X, there exists a pair of open sets, one containing x but not y, and the other containing y but not x.

Definition 2.8 [12]: A space (X,τ) is called semi -T_0 (briefly written as s-T_0), if and only if, each pair of distinct points x, y of X, there exist a semi open set containing one but not the other

Definition 2.9 [12]: A space (X,τ) is called semi-$T_{1/2}$ (briefly written as s-$T_{1/2}$), if and only if, every semi generalized closed set is semi closed.

Definition 2.10 [12]: A space (X,τ) is called semi-T_1 (briefly written as s-T_1), if and only if, each pair of distinct points x, y of X, there exists a pair of semi open sets, one containing x but not y and the other containing y but not x.

Definition 2.11 [9]: Let U is the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then it satisfies the following axioms:

- U and $\Phi \in \tau_R(X)$
- The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$
- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$

Then $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. $(U, \tau_R(X))$ is called the nano topological space. Elements of the nano topology are known as nano open sets in U. Elements of $[\tau_R(X)]^C$ are called nano closed sets in $\tau_R(X)$.

Definition 2.12 [9]: Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be

- Nano semi open if $A \subseteq Ncl[NInt(A)]$

- Nano semi closed if $NInt[Ncl(A)] \subseteq A$

NSO (U, X), NSC (U, X) respectively denote the families of all nano semi open, nano semi closed subsets of U.

Definition 2.13 [3]: If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, Then

- The nano semi-closure of the set A is defined as the intersection of all nano semi closed sets containing A and it is denoted by Nscl (A). Nscl (A) is the smallest nano semi closed set containing A.
- The nano semi-interior of the set A is defined as the union of all nano semi open subsets contained in A and it is denoted by NsInt (A). NsInt (A) is the largest nano semi open subset of A.

Definition 2.14 [3]: A subset A of $(U, \tau_R(X))$ is called nano semi-generalized closed set (briefly Nsg-closed) if $Nscl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano semi open in $(U, \tau_R(X))$.

Definition 2.15 [5]: Let $(X, \tau_{1,2})$ be a bitopological space and $A \subseteq U$. Then A is said to be

- (1, 2)* Semi open if $A \subseteq \tau_{1,2}cl[\tau_{1,2}Int(A)]$
- (1, 2)* Semi closed if $\tau_{1,2}Int[\tau_{1,2}cl(A)] \subseteq A$

(1, 2)*SO(X), (1, 2)*SC(X) respectively denote the families of all (1, 2)* semi open, (1, 2)* semi closed subsets of X.

Definition 2.16 [5]: If $(X, \tau_{1,2})$ is a bitopological space with respect to X and if $A \subseteq X$, then

- The (1, 2)* semi-closure of the set A is defined as the intersection of all (1, 2)* semi closed sets containing A and it is denoted by $\tau_{1,2}$ scl(A). $\tau_{1,2}$ scl (A) is the smallest (1, 2)* semi closed set containing A.
- The (1, 2)* semi-interior of the set A is defined as the union of all (1, 2)* semi open subsets of A contained in A and it is denoted by $\tau_{1,2}$ sInt (A). $\tau_{1,2}$ sInt (A) is the largest (1, 2)*semi open subset of A.

Definition 2.17 [5] A subset A of $(X, \tau_{1,2})$ is called (1, 2)* semi-generalized closed set (briefly (1, 2)* sg-closed) if $\tau_{1,2}scl(A) \subseteq U$ whenever $A \subseteq U$ and U is (1, 2)* semi open in $(X, \tau_{1,2})$.

3. NANO (1, 2)* SEMI-GENERALIZED CLOSED SETS

In this section, the definition of nano bitopological space and nano (1, 2)* semi-generalized closed sets are introduced and studied some of its properties.

Definition 3.1: Let U be the universe, R be an equivalence relation on U and $\tau_{R_{1,2}}(X) = \cup\{\tau_{R_1}(X), \tau_{R_2}(X)\}$ where $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ and $X \subseteq U$. Then it is satisfies the following axioms:

- U and $\Phi \in \tau_R(X)$
- The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $(U, \tau_{R_{1,2}}(X))$ is called the nano bitopological space. Elements of the nano bitopology are known as nano (1, 2)* open sets in U. Elements of $[\tau_{R_{1,2}}(X)]^c$ are called nano (1, 2)* closed sets in $\tau_{R_{1,2}}(X)$.

Example 3.2: Let $U = \{a,b,c,d\}$ with $U / R = \{\{c\},\{d\},\{a,b\}\}$

$X_1 = \{a,c\}$ And $\tau_{R_1}(X) = \{U, \phi, \{c\}, \{a,b,c\}, \{a,b\}\}$

$X_2 = \{b,d\}$ And $\tau_{R_2}(X) = \{U, \phi, \{d\}, \{a,b,d\}, \{a,b\}\}$

Then $\tau_{R_{1,2}}(X) = \{U, \phi, \{c\}, \{d\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$ are (1, 2)* open sets.

The nano (1, 2)* closed sets = $\{U, \phi, \{c\}, \{d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}\}$.

Definition 3.3: If $(U, \tau_{R_{1,2}}(X))$ is a nano bitopological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

- The nano (1, 2)* closure of A is defined as the intersection of all nano (1, 2)* closed sets containing A and it is denoted by $N\tau_{1,2}cl(A)$. $N\tau_{1,2}cl(A)$ is the smallest nano (1, 2)* closed set containing A.
- The nano (1, 2)* interior of A is defined as the union of all nano (1, 2)* open subsets of A contained in A and it is denoted by $N\tau_{1,2}Int(A)$. $N\tau_{1,2}Int(A)$ Is the largest nano (1, 2)* open subset of A.

Definition 3.4: A subset A of $(U, \tau_{R_{1,2}}(X))$ is called nano (1, 2)* semi open set if $A \subseteq N\tau_{1,2}cl[N\tau_{1,2}Int(A)]$. The complement of a nano (1, 2)* semi open set of a space U is called nano (1, 2)* semi closed set in $(U, \tau_{R_{1,2}}(X))$.

Theorem 3.5: Let $(U, \tau_{R_{1,2}}(X))$ be a nano bitopological space. If a subset A of a nano bitopological space

$(U,\tau_{R_{1,2}}(X))$ is nano (1, 2)* closed set in $(U,\tau_{R_{1,2}}(X))$, then A is a nano (1, 2)* semi closed set in $(U,\tau_{R_{1,2}}(X))$.

Proof: Let A be a nano (1, 2)* closed set. That is $N\tau_{1,2}cl(A)=A$. To prove $N\tau_{1,2}Int(N\tau_{1,2}cl(A))\subseteq A$ Since A is a nano (1, 2)* semi closed set. Implies $N\tau_{1,2}Int(N\tau_{1,2}cl(A)) = N\tau_{1,2}Int(A)\subseteq A$ (By 3.1(ii)). Hence A is a nano (1, 2)* semi closed set. Also every nano (1, 2)* open set is nano (1, 2)* semi open set.

Example 3.6: Let $U=\{a,b,c,d\}$ with $U/R=\{\{c\},\{d\},\{a,b\}\}$

$X_1=\{a,c\}$ And $\tau_{R_1}(X)=\{U,\phi,\{c\},\{a,b,c\},\{a,b\}\}$

$X_2=\{b,d\}$ And $\tau_{R_2}(X)=\{U,\phi,\{d\},\{a,b,d\},\{a,b\}\}$

Then $\tau_{R_{1,2}}(X)=\{U,\phi,\{c\},\{d\},\{a,b\},\{a,b,c\},\{a,b,d\}\}$ are (1, 2)*open sets.

The nano (1, 2)* closed sets = $\{U,\phi,\{c\},\{d\},\{c,d\},\{a,b,c\},\{a,b,d\}\}$.

The nano (1, 2)* semi closed sets = $\{U,\phi,\{c\},\{d\},\{a,b\},\{c,d\},\{a,b,c\},\{a,b,d\}\}$

Let A={c, d} be a nano (1, 2)* closed set.

$N\tau_{1,2}cl(A)=\{c,d\}$.

$N\tau_{1,2}Int(N\tau_{1,2}cl(A))=\{c,d\}$ Which implies $N\tau_{1,2}Int(N\tau_{1,2}cl(A))\subseteq A$

Hence every nano (1, 2)* closed set is a nano (1, 2)* semi closed set.

Remark 3.7: The converse of the Theorem 3.5 is not true. In the Example 3.6, {a, b} is a nano (1, 2)* semi closed set but it is not nano (1, 2)* closed set.

Definition 3.8: If $(U,\tau_{R_{1,2}}(X))$ is a nano bitopological space with respect to X where $X\subseteq U$ and if $A\subseteq U$, then

- The nano (1, 2)* semi-closure of A is defined as the intersection of all nano (1, 2)* semi closed sets containing A and it is denoted by $N\tau_{1,2}scl(A)$. $N\tau_{1,2}scl(A)$ is the smallest nano (1, 2)* semi closed set containing A.
- The nano (1, 2)* semi-interior of A is defined as the union of all nano (1, 2)* semi open subsets of A contained in A and it is denoted by $N\tau_{1,2}sInt(A)$. $N\tau_{1,2}sInt(A)$ Is the largest nano (1, 2)* semi open subsets of A.

Definition 3.9: A subset A of $(U,\tau_{R_{1,2}}(X))$ is called nano (1, 2)* semi-generalized closed set (briefly

N (1, 2)*sg-closed) if $N\tau_{1,2}scl(A)) \subseteq V$ whenever $A \subseteq V$ and V is nano (1, 2)* semi open in $(U, \tau_{R_{1,2}}(X))$

Example 3.10: Let $U = \{a, b, c, d\}$ with $U / R = \{\{c\}, \{d\}, \{a, b\}\}$

$X_1 = \{a, c\}$ And $\tau_{R_1}(X) = \{U, \phi, \{c\}, \{a, b, c\}, \{a, b\}\}$

$X_2 = \{b, d\}$ And $\tau_{R_2}(X) = \{U, \phi, \{d\}, \{a, b, d\}, \{a, b\}\}$

Then $\tau_{R_{1,2}}(X) = \{U, \phi, \{c\}, \{d\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ which are (1, 2)* open sets.

The nano (1, 2)* closed sets = $\{U, \phi, \{c\}, \{d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}\}$.

The nano (1, 2)* semi closed sets = $\{U, \phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}\}$

The nano (1, 2)* semi open sets = $\{U, \phi, \{a, b, d\}, \{a, b, c\}, \{c, d\}, \{a, b\}, \{d\}, \{c\}\}$

The nano (1, 2)* semi-generalized open sets are

$\{U, \phi, \{a\}\{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\},$

$\{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$.

The nano (1, 2)* semi-generalized closed sets are

$\{U, \phi, \{a\}\{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\},$

$\{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$.

Theorem 3.11: Let $(U, \tau_{R_{1,2}}(X))$ be a nano bitopological space. If a subset A of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is nano (1, 2)* closed set in $(U, \tau_{R_{1,2}}(X))$, then A is a nano (1, 2)* semi-generalized closed set in $(U, \tau_{R_{1,2}}(X))$.

Proof: Let A be a nano (1, 2)* closed set in U and $A \subseteq V$, V is nano (1, 2)* semi open in U. Since A is nano (1,2)* closed, $N\tau_{1,2}cl(A) = A$. So, $A \subseteq V$ and $N\tau_{1,2}cl(A) = A$ imply $N\tau_{1,2}cl(A) \subseteq V$. Also, $N\tau_{1,2}scl(A) \subseteq N\tau_{1,2}cl(A)$ implies $N\tau_{1,2}scl(A) \subseteq V$, $A \subseteq V$, V is nano (1, 2)* semi open in U. Therefore, A is a nano (1, 2)* semi-generalized closed set.

Theorem 3.12: Let $(U, \tau_{R_{1,2}}(X))$ be a nano bitopological space. If a subset A of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is nano (1, 2)* semi closed set in $(U, \tau_{R_{1,2}}(X))$, then A is a nano (1, 2)* semi-generalized closed set in

$(U, \tau_{R_{1,2}}(X))$.

Proof: Let A be a nano (1, 2)* semi closed set of U and $A \subseteq V$, V is nano (1, 2)* semi open in U. Since A is nano (1, 2)* semi closed, $N\tau_{1,2}scl(A) = A$. So, $A \subseteq V$ and $N\tau_{1,2}scl(A) = A$ imply $N\tau_{1,2}scl(A) \subseteq V$. Also, $A \subseteq V$, V is nano (1, 2)* semi open in U. Therefore, A is a nano (1, 2)* semi-generalized closed set.

Remark 3.13: The converse of the above Theorem 3.12 need not be true. In the Example 3.10, let $A = \{b, c, d\} \subseteq U$. Here $N\tau_{1,2}scl(A) \subseteq V$, $A \subseteq V$, V is nano (1, 2)* semi open set in U. Hence A is nano (1, 2)* semi-generalized closed set. Now $N\tau_{1,2}cl(A) = U$ and $N\tau_{1,2}Int(N\tau_{1,2}cl(A)) = U \not\subseteq A$ which implies that the set A is not a nano (1, 2)* semi closed set.

Theorem 3.14: The intersection of two nano (1, 2)* semi-generalized closed sets in $(U, \tau_{R_{1,2}}(X))$ is also a nano (1, 2)* semi-generalized closed set in $(U, \tau_{R_{1,2}}(X))$.

Proof: Let A and B be two nano (1, 2)* semi-generalized closed sets in $(U, \tau_{R_{1,2}}(X))$. Let V be a nano (1, 2)* semi open set in U such that $A \subseteq V$ and $B \subseteq V$. Then $A \bigcap B \subseteq V$ As A and B are nano (1, 2)* semi-generalized closed sets in $(U, \tau_{R_{1,2}}(X))$, $N\tau_{1,2}scl(A) \subseteq V$ and $N\tau_{1,2}scl(B) \subseteq V$. Now $N\tau_{1,2}scl(A \bigcap B) = N\tau_{1,2}scl(A) \bigcap N\tau_{1,2}scl(B) \subseteq V$ Thus we have $N\tau_{1,2}scl(A \bigcap B) \subseteq V$ whenever $A \bigcap B \subseteq V$, V is nano (1, 2)* semi open set in $(U, \tau_{R_{1,2}}(X))$ which implies $A \bigcap B$ is a nano (1, 2)* semi-generalized closed set in $(U, \tau_{R_{1,2}}(X))$.

Example 3.15: Let $U = \{a, b, c, d\}$ with

$U / R_1 = \{\{a\}, \{d\}, \{b, c\}\}$, $X_1 = \{b, c\}$ and $\tau_{R_1}(X) = \{U, \phi, \{b, c\}\}$

$U / R_2 = \{\{a\}, \{c\}, \{b, d\}\}$, $X_2 = \{b, d\}$ and $\tau_{R_2}(X) = \{U, \phi, \{b, d\}\}$

Then $\tau_{R_{1,2}}(X) = \{U, \phi, \{b, c\}, \{b, d\}\}$ are (1, 2)* open sets.

The nano (1,2)* semi open sets are
$\{U, \phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$.

Then the nano (1, 2)* semi-generalized closed sets are

$\{U, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, c, d\}\}$.

Here $\{a,c,d\}\cap\{a,c\}=\{a,c\}$ is also a nano (1, 2)* semi-generalized closed Sets.

Theorem 3.16: Let A be a nano (1,2)* semi-generalized closed subset of $(U,\tau_{R_{1,2}}(X))$. If $A\subseteq B\subseteq N\tau_{1,2}scl(A)$, then B is also a nano (1, 2)* semi-generalized closed subset of $(U,\tau_{R_{1,2}}(X))$

Proof: Let V be a nano (1, 2)* semi open set of a nano (1, 2)* semi-generalized closed subset of $\tau_{R_{1,2}}(X)$ such that $B\subseteq V$. As $A\subseteq B$, that is $A\subseteq V$. As A is a nano (1, 2)* semi-generalized closed set, $N\tau_{1,2}scl(A)\subseteq V$. Given $B\subseteq N\tau_{1,2}scl(A)$, then $N\tau_{1,2}scl(B)\subseteq N\tau_{1,2}scl(A)$. As $N\tau_{1,2}scl(B)\subseteq N\tau_{1,2}scl(A)$ and $N\tau_{1,2}scl(A)\subseteq V$, which implies $N\tau_{1,2}scl(B)\subseteq V$ whenever $B\subseteq V$ and V is nano (1, 2)* semi open. Hence B is also a nano (1, 2)* semi-generalized closed subset of $(U,\tau_{R_{1,2}}(X))$.

Theorem 3.17: Let A be a nano (1, 2)* semi generalized closed set in $(U,\tau_{R_{1,2}}(X))$. Then $N\tau_{1,2}scl(A)-A$ has no non-empty nano (1, 2)* semi closed set.

Proof: Let A be nano (1, 2)* semi generalized closed set in $(U,\tau_{R_{1,2}}(X))$ and F be a nano (1, 2)* semi closed subset of $N\tau_{1,2}scl(A)-A$. That is, $F\subseteq N\tau_{1,2}scl(A)-A$. Which implies that $F\subseteq N\tau_{1,2}scl(A)\cap A^c$. That is $F\subseteq N\tau_{1,2}scl(A)$ and $F\subseteq A^c$. $F\subseteq A^c$ Implies that $A\subseteq F^c$ where F^c is a nano (1, 2)* semi open set. Since A is nano (1,2)* semi generalized closed, $N\tau_{1,2}scl(A)\subseteq F^c$ That is, $F\subseteq[N\tau_{1,2}scl(A)]^C$. Thus $F\subseteq N\tau_{1,2}scl(A)\cap[N\tau_{1,2}scl(A)]^C=\phi$ Therefore $F=\phi$

Theorem 3.18: Let A be a nano (1, 2)* semi generalized closed set in $(U,\tau_{R_{1,2}}(X))$. Then A is nano (1, 2)* semi closed if and only if, $N\tau_{1,2}scl(A)-A$ is nano (1, 2)* semi closed set.

Proof: Let A be a nano (1, 2)* semi generalized closed set in $(U,\tau_{R_{1,2}}(X))$. If A is nano (1, 2)* semi closed, then $N\tau_{1,2}scl(A)-A=\phi$, which is a nano (1, 2)* semi closed set.

Conversely, let $N\tau_{1,2}scl(A)-A$ be nano (1,2)* semi closed. Then by the above Theorem 3.17 $N\tau_{1,2}scl(A)-A$ does not contain any non-empty nano (1,2)* semi closed set. Thus, $N\tau_{1,2}scl(A)-A=\phi$. That is, $N\tau_{1,2}scl(A)=A$. Therefore A is nano (1, 2)* semi closed.

4. CHARACTERIZATIONS ON NANO (1, 2)* SEMI-GENERALIZED SPACES

In this section some new characterizations of Nano (1, 2)* semi-generalized spaces are introduced and studied some of its properties.

Definition 4.1: If $(U, \tau_{R_{1,2}}(X))$ is a nano bitopological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, Then

- The nano (1, 2)*semi generalized closure of A is defined as the intersection of all nano (1, 2)* semi generalized closed sets containing A and it is denoted by $N\tau_{1,2}sgcl(A)$. $N\tau_{1,2}sgcl(A)$ is the smallest nano (1, 2)* semi generalized closed set containing A.
- The nano (1, 2)* semi generalized interior of A is defined as the union of all nano (1, 2)* semi generalized open subsets of A contained in A and it is denoted by $N\tau_{1,2}sgInt(A)$ $N\tau_{1,2}sgInt(A)$ is the largest nano (1,2)* semi generalized open subset of A.

Definition 4.2: A space $(U, \tau_{R_{1,2}}(X))$ is called nano (1,2)*-T_0 (briefly written as N (1, 2)*-T_0), if and only if, each pair of distinct points x, y of $\tau_{R_{1,2}}(X)$, there exist a nano (1, 2)* open set containing one but not the other.

Definition 4.3: A space $(U, \tau_{R_{1,2}}(X))$ is called nano (1,2)* semi -T_0 (briefly written as N(1, 2)*s-T_0), if and only if, each pair of distinct points x, y of $\tau_{R_{1,2}}(X)$, there exist a nano (1, 2)* semi open set containing one but not the other.

Definition 4.4: A space $(U, \tau_{R_{1,2}}(X))$ is called nano (1,2)* semi-generalized-T_0 (briefly written as N (1,2)*sg-T_0), if and only if, each pair of distinct points x, y of $\tau_{R_{1,2}}(X)$, there exist a nano (1, 2)* semi generalized open set containing one but not the other.

Remark 4.5: Every nano (1,2)* semi-T_0 space is nano (1,2)* semi generalized-T_0 space since every nano (1,2)* semi open set is nano (1,2)* semi generalized open set but converse is not true.

Theorem 4.6: If in any nano bitopological space $(U, \tau_{R_{1,2}}(X))$, nano (1, 2)* semi generalized closures of distinct points are distinct, then $\tau_{R_{1,2}}(X)$ is nano (1, 2)* semi generalized- T_0

Proof: Let $x, y \in \tau_{R_{1,2}}(X)$, $x \neq y$ imply $N\tau_{1,2}sgcl\{x\} \neq N\tau_{1,2}sgcl\{y\}$. Then there exists a point $z \in \tau_{R_{1,2}}(X)$ such that z belongs one of two sets, say $N\tau_{1,2}sgcl\{y\}$ but not to $N\tau_{1,2}sgcl\{x\}$. If suppose that $z \in N\tau_{1,2}sgcl\{x\}$, then $z \in N\tau_{1,2}sgcl\{y\} \subset N\tau_{1,2}sgcl\{x\}$, which is contradiction. So, $y \in X - N\tau_{1,2}sgcl\{x\}$, where $X - N\tau_{1,2}sgcl\{x\}$ is nano (1, 2)* semi generalized open set which does not contain x, This shows that X is semi generalized-T_0

Theorem 4.7: In any nano bitopological space $(U, \tau_{R_{1,2}}(X))$, nano (1, 2)* semi generalized closures of distinct points are distinct.

Proof: Let $x, y \in \tau_{R_{1,2}}(X)$ with $x \neq y$. To show that $N\tau_{1,2}sgcl\{x\} \neq N\tau_{1,2}sgcl\{y\}$ Considered the two sets $N\tau_{1,2}sgcl\{x\}$ and $N\tau_{1,2}sgcl\{y\}$ in $\tau_{R_{1,2}}(X)$. Then there exists a point $z \in \tau_{R_{1,2}}(X)$ such that z belongs one of two sets, say $N\tau_{1,2}sgcl\{y\}$ but not to $N\tau_{1,2}sgcl\{x\}$. If suppose that $z \in N\tau_{1,2}sgcl\{x\}$, then $z \in N\tau_{1,2}sgcl\{y\} \subset N\tau_{1,2}sgcl\{x\}$, which is contradiction. Hence $N\tau_{1,2}sgcl\{x\} \neq N\tau_{1,2}sgcl\{y\}$

Definition 4.8: A space $(U, \tau_{R_{1,2}}(X))$ is called nano (1, 2)*-$T_{1/2}$ (briefly written as N (1, 2)*- $T_{1/2}$), if and only if, every nano (1, 2)* generalized closed set is nano (1, 2)* closed.

Definition 4.9: A space $(U, \tau_{R_{1,2}}(X))$ is called nano (1, 2)* semi-$T_{1/2}$ (briefly written as N (1, 2)* s-$T_{1/2}$), if and only if, every nano (1, 2)* semi generalized closed set is nano (1, 2)* semi closed.

Theorem 4.10: A space $(U, \tau_{R_{1,2}}(X))$ is nano (1, 2)* semi-$T_{1/2}$ space. Then for each $x \in \tau_{R_{1,2}}(X)$, {x} is nano (1, 2)* semi open or nano (1, 2)* semi closed.

Proof: Suppose that for some $x \in \tau_{R_{1,2}}(X)$, {x} is not nano (1, 2)* semi closed. Since $\tau_{R_{1,2}}(X)$ is the only nano (1, 2)* semi open set containing $\{x\}^c$, the set $\{x\}^c$ is nano (1, 2)* semi generalized closed and so it is nano (1, 2)* semi closed in the nano (1, 2)* semi-$T_{1/2}$ space $(U, \tau_{R_{1,2}}(X))$. Therefore {x} is nano (1, 2)* semi open

Theorem 4.11: A space $(U, \tau_{R_{1,2}}(X))$ is nano (1,2)* semi-$T_{1/2}$, if and only if, every subset of X is the intersection of all nano (1, 2)* semi open sets and all nano (1, 2)* semi closed sets containing it.

Proof: Necessity: Let $(U, \tau_{R_{1,2}}(X))$ be nano (1, 2)* semi-$T_{1/2}$ space with $B \subset X$ arbitrary. Then $B = \bigcap \left\{ \{x\}^c : x \notin B \right\}$ is an intersection of nano (1, 2)* semi open sets and nano (1, 2)* semi closed sets by Theorem 4.10, it is nano (1, 2)* semi open sets and nano (1, 2)* semi closed sets.

Sufficiency:

For each $x \in X$, $\{x\}^c$ is the intersection of all nano (1, 2)* semi open sets and all nano (1, 2)* semi closed sets containing it. Thus $\{x\}^c$ is either nano (1, 2)* semi open or nano (1, 2)* semi closed and hence X is nano (1, 2)* semi-$T_{1/2}$

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Definition 4.12: A space $(U, \tau_{R_{1,2}}(X))$ is called nano (1, 2)*-T_1 (briefly written as N (1, 2)*- T_1), if and only if, each pair of distinct points x, y of X, there exists a pair of nano (1, 2)* open sets, one containing x but not y, and the other containing y but not x.

Definition 4.13: A space $(U, \tau_{R_{1,2}}(X))$ is called nano (1, 2)* semi-T_1 (briefly written as N (1, 2)*s- T_1), if and only if, each pair of distinct points x, y of X, there exists a pair of nano (1, 2)* semi open sets, one containing x but not y and the other containing y but not x.

Definition 4.14: A space $(U, \tau_{R_{1,2}}(X))$ is called nano (1, 2)* semi generalized -T_1 (briefly written as N(1,2)*sg- T_1), if and only if, each pair of distinct points x, y of X, there exists a pair of nano (1, 2)* semi generalized open sets, one containing x but not y, and the other containing y but not x.

Theorem 4.15: Let $(U, \tau_{R_{1,2}}(X))$ be a nano (1, 2)* semi symmetric space. Then the following are equivalent

- $(U, \tau_{R_{1,2}}(X))$ is nano (1, 2)* semi-T_0
- $(U, \tau_{R_{1,2}}(X))$ is nano (1, 2)* semi-$T_{1/2}$
- $(U, \tau_{R_{1,2}}(X))$ is nano (1, 2)* semi-T_1

Proof: $(i) \rightarrow (ii)$ Let A be a subset of $(U, \tau_{R_{1,2}}(X))$ is nano (1, 2)* semi-T_0 by Remark 4.5 Every nano (1.2)* semi-T_0 space is nano (1.2)* semi generalized-T_0 space.

Hence A is is nano (1, 2)* semi-$T_{1/2}$ in $(U, \tau_{R_{1,2}}(X))$.

$(ii) \rightarrow (iii)$ Let A be a subset of $(U, \tau_{R_{1,2}}(X))$ is nano (1, 2)* semi-$T_{1/2}$. That is A is semi generalized closed. By theorem 4.7 A is nano (1, 2)* semi-T_1 in $(U, \tau_{R_{1,2}}(X))$

$(iii) \rightarrow (i)$ Let $x \neq y$ and since $(U, \tau_{R_{1,2}}(X))$ is nano (1, 2)* semi-T_0. Let as assume that $x \in N\tau_{1,2}O \subset \{y\}^c$ for some $N\tau_{1,2}O \in SO(U, \tau_{R_{1,2}}(X))$. Then $x \notin N\tau_{1,2}scl\{y\}$ and hence $y \notin N\tau_{1,2}scl\{x\}$. Therefore there exists $N\tau_{1,2}O_1 \in SO(U, \tau_{R_{1,2}}(X))$ such that $y \in N\tau_{1,2}O \subset \{x\}^c$ and $(U, \tau_{R_{1,2}}(X))$ is a nano (1,2)* semi-T_1 space.

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